Non-classical state stabilization in a cavity by reservoir engineering

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Abstract: We propose an engineered reservoir inducing the relaxation of a cavity field towards non-classical states. It is made up of two-level atoms crossing the cavity one at a time. Each atom-cavity interaction is first dispersive, then resonant, then dispersive again. The reservoir pointer states are those produced by a fictitious Kerr Hamiltonian acting on a coherent field. We thereby stabilize squeezed states and quantum superpositions of multiple coherent components in a cavity having a finite damping time. This robust method could be implemented in state-of-the-art experiments and lead to interesting insights into mesoscopic quantum state superpositions.

Non-classical states of the radiation field are the focus of a considerable interest. Squeezed states (SS), with reduced fluctuations on one field quadrature, are interesting for high precision quantum measurements [1]. Mesoscopic field state superpositions (MFSS), involving coherent components with different classical properties, are reminiscent of the famous Schrödinger cat [2], in a superposition of the "dead" and "alive" states. Their environment-induced decoherence sheds light on the quantum-classical boundary [3]. Our aim is to generate and even stabilize such states using reservoir engineering

Many experiments on MFSS have been proposed or realized, particularly with trapped ions [4] (whose harmonic motion is equivalent to a field mode) or cavity quantum electrodynamics (CQED) [3], with a single atom coupled to a single field mode. Introducing the atom in a state superposition and finally detecting it leads to the preparation of a MFSS, conditionned by the atomic detection outcome [3, 5, 6, 7, 8, 9, 10, 11].

Deterministic preparation of MFSS could, in principle, be achieved by propagation of a coherent field in a Kerr medium [12], described by the Hamiltonian:

$$H_K = \zeta_K \, \mathbf{N} + \gamma_K \, \mathbf{N}^2 \; , \tag{1}$$

($\mathbf{N}$: photon number operator ; ζ_K is proportional to the linear index ; γ_K: Kerr frequency; units are chosen such that $\hbar = 1$ throughout the paper). An initial coherent state $|\alpha\rangle$ evolves with the interaction time t_K through a sequence of nonclassical states $e^{-i\, t_K H_K}|\alpha\rangle$ distributed in phase space along the circle with radius $|\alpha|$ [3, 7.2]. For $t_K\gamma_K \ll \pi$, the field is in a quadrature-squeezed state $|s_\alpha\rangle$ with a nearly Gaussian Wigner function W. For $t_K\gamma_K = \pi/k$, we get a MFSS $|k_\alpha\rangle$ of k equally spaced coherent components. For $t_K\gamma_K = \pi/2$, a "Schrödinger cat" state $|c_\alpha\rangle = (|\alpha e^{i\varphi_\alpha}\rangle + i\,|\text{-}\alpha e^{i\varphi_\alpha}\rangle)/\sqrt{2}$ is reached (with $\varphi_\alpha = -\zeta_K t_K$). Note that the collisional interaction Hamiltonian for an atomic sample in an optical lattice is similar to H_K [13].

The unconditional *preparation*, *protection* and long-term *stabilization* of SS and MFSS is an essential goal for the study and practical use of these states. We envision in this Letter a reservoir engineering setup in CQED that would drive to e.g. $|c_\alpha\rangle$ *any* initial state.

Reservoir engineering [14, 15] protects target quantum states by coupling the system to an "engineered" bath whose pointer states [16] include the target. The system is effectively decoupled from its standard environment by its much stronger coupling with the engineered bath.

For trapped ions, reservoirs composed of laser fields can stabilize a subspace containing superpositions of coherent vibrations [14, 11]. However, they do not prevent mixing of states belonging to the stabilized subspace, making it impossible to protect a specific MFSS [3, pp. 487-488]. A reservoir stabilizing the components on n vibration states with $n + 2$ lasers is proposed in [15]. In CQED proposals, reservoirs protect squeezed states [17] and entanglement of two field modes [18]. In [19], a reservoir made up of a stream of atoms crossing the cavity stabilizes MFSS (cotangent states). However, the scheme is based on the trapping state condition [20] (a resonant atom entering the cavity in its upper state undergoes a $2p\pi$ quantum Rabi pulse in an n-photon field), very sensitive to finite cavity temperature.

We propose a robust method to stabilize SS and MFSS in a CQED experiment. The engineered reservoir is made up of a stream of 2-level atoms undergoing a tailored

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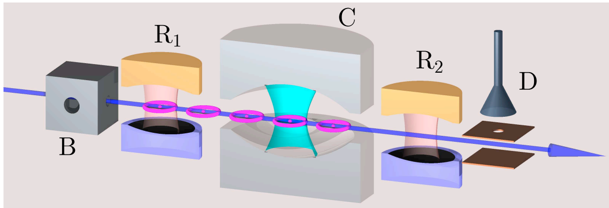

Figure 1: Scheme of the ENS CQED experiment.

composite interaction with the cavity, combining resonant and dispersive parts. The reservoir pointer states are $\approx e^{-i\,t_K H_K}|\alpha\rangle$, in which α and $\gamma_K t_K$ can be chosen at will. It stabilizes the states $|s_\alpha\rangle$, $|k_\alpha\rangle$ and $|c_\alpha\rangle$.

For the sake of definiteness, we consider the ENS CQED set-up (Fig. 1, details in [3, 7]). The microwave field is trapped in the cavity C (resonance frequency $\omega_c/2\pi =$ 51 GHz), made up of highly reflecting superconducting mirrors cooled at 0.8 K (mean number of blackbody photons per mode $n_t = 0.05$). The photon lifetime in C is $T_c = 0.13$ s.

The field in C is controlled through its interaction with circular Rydberg states. Atomic samples are produced at regular time intervals in box B out of a rubidium atomic beam. They have an adjustable velocity v. The probability for having one atom in a sample, $p_{at} \simeq 0.3$, is kept low to avoid simultaneous interaction of two atoms with C. The transition between circular Rydberg states of principal quantum number 50 ($|g\rangle$) and 51 ($|e\rangle$) at frequency $\omega_0/2\pi$ is nearly resonant with C. The atom-cavity detuning $\delta = \omega_0 - \omega_c \ll \omega_c$ can be controlled with an excellent time resolution by ways of the Stark effect produced by a static field applied across cavity mirrors.

Atoms undergo a Ramsey pulse in zone R_1, preparing them in an adjustable state $|(u,\eta)_{\rm at}\rangle = \cos(u/2)|g\rangle + e^{i\eta}\sin(u/2)|e\rangle$. They then interact with C and generally get entangled with its field. The atomic state is finally measured in an arbitrary basis by a combination of a second Ramsey zone R_2 and a state-resolving field-ionization detector D. For reservoir operation, the final atomic detection is not important. However, it is essential for a complete cavity field quantum state reconstruction [7], performed at the end of the experiment to assess the engineered reservoir action.

Atom-cavity interaction is ruled by the Jaynes-Cummings Hamiltonian:

$$H_{JC} = \frac{\delta}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + i\frac{\Omega(s)}{2}(\,|g\rangle\langle e|\,\mathbf{a}^\dagger - |e\rangle\langle g|\,\mathbf{a}\,) \quad (2)$$

(in a proper interaction representation, $\mathbf{a}$ photon annihilation operator). The atom-field coupling is measured by $\Omega(s)$ at position s along the atomic trajectory. In the Gaussian mode of C at time t, $\Omega(s) = \Omega_0\; e^{-s^2/w^2}$ with $s = vt$, $\Omega_0/2\pi = 50$ kHz and $w = 6$ mm (the origin of space and time is when the atom crosses cavity axis).

The engineered reservoir uses atoms performing a combination of resonant ($\delta = 0$) and dispersive ($|\delta| > \Omega_0$) interactions. The corresponding unitary evolution operators (under adiabatic approximation for $|\delta| > \Omega_0$) are, within irrelevant phases [3]:

$$\begin{aligned} U_r \quad (\Theta) &= |g\rangle\langle g|\,\cos(\Theta\sqrt{\mathbf{N}}/2) + |e\rangle\langle e|\,\cos(\Theta\sqrt{\mathbf{N+1}}/2) \\ &\quad -|e\rangle\langle g|\,\frac{\sin(\Theta\sqrt{\mathbf{N+1}}/2)}{\sqrt{\mathbf{N+1}}}\,\mathbf{a} + |g\rangle\langle e|\,\mathbf{a}^\dagger\,\frac{\sin(\Theta\sqrt{\mathbf{N+1}}/2)}{\sqrt{\mathbf{N+1}}} \end{aligned} \quad (3)$$

$$U_d \quad (\phi(\mathbf{N})) = |g\rangle\langle g|\;e^{-i\,\phi(\mathbf{N})} + |e\rangle\langle e|\;e^{+i\,\phi(\mathbf{N+1})}\;, \qquad (4)$$

where $\Theta = \int (\Omega(s)/v)\,ds$ is the integrated quantum Rabi pulsation in vacuum. The dispersive phase $\phi(\mathbf{N})$ is diagonal in the Fock state basis $\{|n\rangle\}$, with components $\phi(n) = -\delta/(2v)\int(\sqrt{1+n\,(\Omega(s)/\delta)^2}-1)ds$, reducing to $\phi(n) = -n/(4v\,\delta)\int\Omega^2(s)ds$ in the $|\delta| \gg \Omega$ limit.

Atoms entering C in $|g\rangle$ and undergoing resonant interaction with it constitute a reservoir stabilizing the vacuum state $|0\rangle$. Similarly, atoms entering C in $|(u,\eta)_{\rm at}\rangle$, $u \ll 1$, for a resonant interaction with $\Theta \ll 1$, stabilize in very good approximation a coherent state $|\alpha\rangle$ with $\alpha = 2ue^{i\eta}/\Theta$, provided u/Θ is of the order of 1 [21]. The cavity is then a 'micromaser' fed by atoms in a coherent superposition [19, 22]. The resulting field has a well-defined phase and a finite amplitude, even if C is undamped, since the atomic medium excitation is below population inversion.

Our key observation is that sandwiching the resonant interaction between two opposite dispersive interactions can be viewed as a basis change on the field, giving access to non-classical pointer states:

$$U_d(\phi(\mathbf{N}))\,U_r(\Theta)\,U_d(-\phi(\mathbf{N})) = e^{-i\,h(\mathbf{N})}U_r(\Theta)e^{i\,h(\mathbf{N})}\;, \qquad (5)$$

where $h(\mathbf{N})$ is a discrete integral of the phase $\phi(\mathbf{N})$, defined by the recurrence relation

$$h(\mathbf{N+1}) - h(\mathbf{N}) = 2\phi(\mathbf{N+1})\;. \qquad (6)$$

We associate $h(0) = 0$ to Eq.(6) without loss of generality, since a nonzero value only reflects on a global phase for the field state. Since the resonant interaction $U_r(\Theta)$ stabilizes the coherent state $|\alpha\rangle$, then $U_d(\phi(\mathbf{N}))U_r(\Theta)U_d(-\phi(\mathbf{N}))$ stabilizes $e^{-i\,h(\mathbf{N})}|\alpha\rangle$. For large detuning [3], $|\delta| \gg \Omega_0$, $\phi(\mathbf{N})$ is linear over the relevant photon numbers: $\phi(\mathbf{N}) \approx \xi_0\mathbf{1} + \xi_1\mathbf{N}$. Then $e^{-i\,h(\mathbf{N})}$ is the evolution operator generated by the Kerr Hamiltonian H_K [Eq.(1)] acting during a time t_K, such that $\gamma_K t_K = \xi_1$ and $\zeta_K t_K = 2\xi_0 + \xi_1$. In particular, for $\xi_1 = \pi/2$, the pointer state $e^{-i\,h(\mathbf{N})}|\alpha\rangle$ is, up to an irrelevant phase, the MFSS $|c_\alpha\rangle = (|\alpha e^{i\varphi_\alpha}\rangle + i\,|\text{-}\alpha e^{i\varphi_\alpha}\rangle)/\sqrt{2}$, with $\varphi_\alpha = -2\xi_0 - \pi/2$. Any non-classical state produced by $e^{-i\,t_K H_K}$ can be stabilized by adjusting ξ_1, i.e. δ.

Let us give an intuitive insight into the stabilization of the 'cat' state $|c_\alpha\rangle$. We take $\eta = 0$, hence α real positive. Let us assume $\phi(\mathbf{N}) = (\pi/2)\mathbf{N}$ i.e. $\varphi_\alpha = -\pi/2$ and an initial cat state $|\psi_0\rangle = (|\text{-}i\alpha_0\rangle + i\,|i\alpha_0\rangle)/\sqrt{2}$ with $\alpha_0 < \alpha$ [23]. The first dispersive interaction entangles the atom and the field, correlating two π-phase shifted atomic dipoles $|(u,0)_{\rm at}\rangle$ and $|(-u,0)_{\rm at}\rangle \equiv |(u,\pi)_{\rm at}\rangle$ with coherent states $|\alpha_0\rangle$ and $|-\alpha_0\rangle$ respectively. Note that this entangled atom-field state is still a mesoscopic quantum superposition. During the resonant interaction, each dipole state amplifies the correlated coherent component from $\pm\alpha_0$ to $\pm\tilde{\alpha}$ ($\alpha_0 < \tilde{\alpha} < \alpha$). The second dispersive interaction disentangles the atom and cavity by reversing the initial entangling operation. The final cavity state is thus independent upon the atomic one and writes $|\psi_t\rangle = (|\text{-}i\tilde{\alpha}\rangle + i\,|i\tilde{\alpha}\rangle)/\sqrt{2}$, a 'larger' MFSS. Similarly, if $\alpha_0 > \alpha$, the atomic interaction reduces the cat amplitude. Altogether, the atoms stabilize a sizeable MFSS in the steady state.

We have performed numerical simulations of the field evolution in realistic experimental conditions. The composite interaction is implemented with a ladder of Stark shifts during cavity crossing by each atomic sample. The atom-field interaction starts when $s = -1.5\,w$ (corresponding to a dispersive coupling equal to $\sim 1\%$ of its maximum value) and ends when $s = 1.5\,w$, the total interaction time being $t_i = 3w/v$. During t_i, δ is first set at $\delta = \Delta > 0$ (first dispersive interaction), then at $\delta = 0$ for a short time span t_r centered on cavity center crossing time, and finally $\delta = -\Delta$ for the second dispersive interaction. The evolution operators corresponding to resonant and dispersive interactions are computed exactly from H_{JC} [Eq. (2)], using the quantum optics package for MATLAB [24] (Hilbert space is truncated to the 60 first Fock states). Cavity relaxation towards thermal equilibrium is taken into account. The atom is finally discarded by tracing the global density matrix over its state. We compute in this way the density matrix ρ describing the field state after each atomic sample. The interaction with the next sample starts immediately after the previous one has left C. Thus, smaller t_i implies more frequent atom-cavity interaction, that is an engineered reservoir dominating more strongly the standard environment.

The steady state is reached after a few tens of interactions only. Figure 2(a)-(f) presents the Wigner functions $W(\xi)$ of the cavity field after its interaction with 200 atomic samples. The (irrelevant) initial field state is the vacuum. Top panels present the results for an ideal experiment, with exactly one atom per sample ($p_{at} = 1$), no cavity damping and no residual thermal field. Bottom panels present the more realistic case of finite cavity damping ($T_c = 0.13$ s, $n_t = 0.05$), with $p_{at} = 0.3$.

Panels (a) and (b) correspond to the two-component MFSS generation, i.e. to the largest Kerr interaction time. To achieve large dispersive interactions with reasonable Δ values ($\Delta = 2.2\,\Omega_0$), we choose slow atomic velocity $v = 70$ m/s still compatible with the horizontal atomic beam in the ENS set-up. This velocity could easily be reached with moderate laser slowing of the atomic beam and yields $t_i = 257\,\mu$s. With $t_r = 5$ μs i.e. $\Theta \approx \pi/2$, and $u = 0.45\pi$, we get a MFSS with strong negativities in its Wigner function. In the ideal case [panel (a)], the final ρ is a pure state with an average photon number $\overline{n} = 2.89$ photons. We estimate its fidelity F with respect to a MFSS of two coherent states with opposite phases, optimized by adjusting in the reference state the phase and amplitude of the coherent components and their relative quantum phase in the superposition. F maximizes at 98% for a reference with $\overline{n} = 2.8$ photons. With relaxation [panel (b)], we get $\overline{n} = 2.72$ photons. The purity $P = Tr(\rho^2)$ is reduced by cavity relaxation down to 51.1%. The fidelity with respect to an ideal cat with $\overline{n} = 2.4$ is 69% and the Wigner function still exhibits negativities revealing non-classical features. We propose to quantify the "quantumness" of this state by $Q = (\mathrm{Wig_i} - \mathrm{Wig_c})/(3\mathrm{Wig_c})$, where $\mathrm{Wig_i}$ is the maximum total amplitude of the fringe pattern between the classical components in W, and $\mathrm{Wig_c}$ is the maximum of W on the classical components. Then $Q = 1$ for a MFSS $|k_\alpha\rangle$ and $Q = 0$ for a statistical mixture. For the cats presented in panels (a) and (b), Q reaches 89% and 76% respectively. In conclusion, we are able to realistically stabilize in the steady state a large 'cat' which would be damped within a decoherence time $T_d = T_c/(2\overline{n}) = 27$ ms without the engineered reservoir.

For a slightly larger detuning, $\Delta = 3.7\,\Omega_0$ (all other parameters unchanged), we obtain a three-component MFSS $|k_\alpha\rangle$ with $k = 3$ [panels (c) and (d)]. In the ideal case, the final pure state has $\overline{n} = 2.86$ photons and fidelity 99% with respect to an ideal MFSS of three coherent components and same energy; quantumness $Q = 91\%$. Adding relaxation, we get a field with $\overline{n} = 2.70$ photons, $P = 56\%$, $F = 73\%$ and $Q = 86\%$.

In the Kerr dynamics, squeezed states are obtained in the early stages of the initial coherent state phase spreading. For large coherent state amplitude and small phase spread, we obtain nearly Gaussian minimal uncertainty states for the Heisenberg relations between orthogonal field quadratures. In our composite pulse simulations, with $v = 300$ m/s so $t_i = 60\,\mu$s, $t_r = 1.7$ μs i.e. $\Theta \approx 0.17\pi$, $u = \pi/2$ and $\Delta = 70\,\Omega_0$, we generate ideally a pure field with $\overline{n} = 32.7$ and a 2.6 dB squeezing. With relaxation, the squeezing is still 1.5 dB and $P = 85\%$, while $\overline{n}$ drops to 21. For larger phase spreads, the Wigner function takes a banana shape. These non-minimal uncertainty states present non-classical negativities. As an example, panels (e) and (f) are obtained with $v = 150$ m/s hence

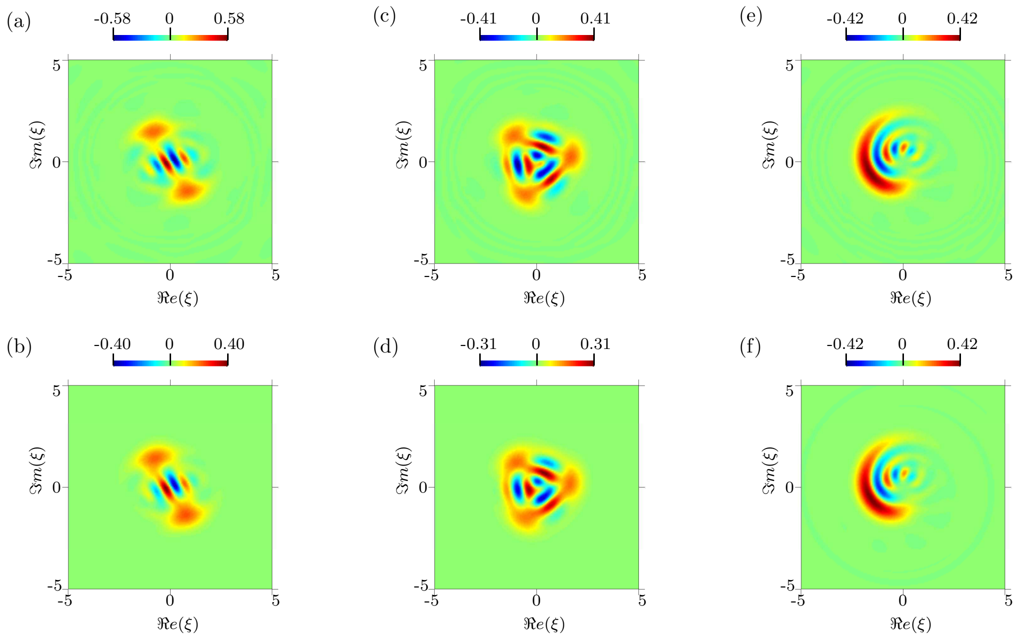

Figure 2: Wigner functions of non-classical states obtained after 200 steps of reservoir-atom interactions, without (top) and with (bottom) coupling to a thermal environment inducing decoherence. (a) and (b): state close to $|c_\alpha\rangle$. (c) and (d): state close to $|k_\alpha\rangle$ with $k = 3$. (e) and (f): state close to $|s_\alpha\rangle$. See the text for detailed conditions.

$t_i = 120\,\mu$s, $t_r = 5\,\mu$s i.e. $\Theta \approx \pi/2$, $u = \pi/2$ and $\Delta = 7\,\Omega_0$. The ideal situation results in a pure field with $\overline{n} = 3.64$, whereas the realistic steady state [panel (f)] has $\overline{n} = 3.52$ and $P = 91\%$. Clearly, the influence of relaxation is reduced with the fast and thus more frequent atoms used here. All these settings are within reach of the present ENS set-up.

We have checked that this scheme is not sensitive to experimental imperfections (a few % variation of the interaction parameters does not appreciably modify the steady state). It does not require recording the final atomic states, unlike quantum feedback experiments, which can also stabilize non-classical states [25]. Feedback could in fact be used in addition to improve the performance of the engineered reservoir, using atomic detection results to detect and react to environment-induced jumps of field state, or to post-select time intervals when a cat is stabilized with high probability [26].

In conclusion, we have shown that the composite atom-cavity interaction scheme realizes an engineered environment for the cavity field, driving it deterministically towards non-classsical field states including the MFSS Schrödinger cat-like states. The scheme is simple and robust enough to be amenable to experiment with a state-of-the-art CQED or circuit QED set-up.

The engineered reservoir, driving *any* initial cavity state to the desired mesoscopic field state superposition and stabilizing these quantum resources for *arbitrarily long times*, opens interesting perspectives for fundamental studies of non-classicality and decoherence.

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